of the improved 3-dB hybrid-ring directional coupler would be better when carefully and elaborately manufactured.

V. Conclusion

A broad-band design theory of the improved 3-dB hybrid-ring directional couplers by CAD was demonstrated, where the concept of a hypothetical port was adopted. Also, the characteristics of the improved 3-dB hybrid ring are compared with those of the conventional rat race and hybrid ring. It was clearly shown that the bandwidth is broadened considerably by dividing the three-quarter-wave equal-admittance section of the conventional hybrid-ring into unequal-admittance sections with proper values, while the symmetry of the circuit is maintained. Hence, the improved hybrid-ring directional coupler can be constructed very easily and its bandwidth reaches up to approximately 50.7 percent. The experiments for two cases as examples were carried out, the results of which agreed well with the numerically designed ones, and, hence, the validity of the broad-band design method was confirmed.

Acknowledgment

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New View on an Anisotropic Medium in a Moving Line Charge Problem

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Abstract—**The electromagnetic field solution of the problem, in which the line charges move uniformly parallel or perpendicular to the interface of two different anisotropic media, is obtained by the method of moving images. The results are shown by representing the moving image line charges for such a problem. A new view on an anisotropic medium in such a problem is discussed by defining the equivalent metric factor and the equivalent normalized metric factor. The minimum principle shown in this short paper states that the electric flux emitted from the moving line charge chooses a trajectory that minimizes the equivalent effective length.**

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I. Introduction

The radiation produced by a uniformly moving point charge has been experimentally discovered by Cerenkov [6] and theoretically investigated by Frank and Tamm [7]. Ginzburg and Frank [8] have shown that a point charge moving uniformly across the interface of two media with different dielectric constants emits a unique radiation, called transition radiation. These radiations have later markedly been investigated by many workers [1]–[5], [9]–[17] (good bibliographies are given in [15] and [20]). The Cerenkov counter (for example, see [4]) and the microwave generator (for example, see [1], [5]) have been considered as the application. On the other hand, the problem of radiation from a source embedded in a moving medium as the inverse problem of the former has been investigated (for example, see [18]–[20], good bibliographies are given in [20]).

From the interests for the boundary value problem in the relativistic electrodynamics, this short paper treats the fields produced by moving charges. The fields produced by charges moving uniformly in two isotropic media have been determined, by the method of moving images, by Beck [9] for the motion perpendicular to the interface and by Sitenko and Tkalich [13] for the motion parallel to the interface.

This short paper derives the solution of electromagnetic fields by the line charges uniformly moving parallel or perpendicular to the interface of two different anisotropic media. This problem may be attacked by the method of moving images for a case that the velocities of all moving line charges are less than the phase velocities of light in those media. Then, the equivalent metric factor and the equivalent normalized metric factor of an anisotropic medium are defined by extending the metric factor and the normalized metric factor in the case of the static problem [21]. The minimum principle of equivalent effective path length for such electric flux is expressed in the form of integration by using the equivalent normalized metric factor.

II. Fields by a Moving Line Charge in a Single Anisotropic Medium

Consider the problem that the line charge λ_0 infinitely extended parallel to the x axis moves uniformly in the positive z-direction with velocity v in a single medium of the following permittivity tensor $\bar{\bar{\epsilon}}$ and permeability μ:

$$\bar{\bar{\epsilon}} = \begin{pmatrix} \epsilon_x^* & 0 & 0 \\ 0 & \epsilon_y^* & 0 \\ 0 & 0 & \epsilon_z^* \end{pmatrix} \epsilon_0 \tag{1}$$

$$\mu = \mu^* \mu_0 \tag{2}$$

where ϵ_x^*, ϵ_y^*, and ϵ_z^* are the relative dielectric constants, μ^* the relative permeability, and ϵ_0 and μ_0 the permittivity and the permeability of vacuum, respectively.

The electromagnetic field is determined by Maxwell's equations for the given line charge density and current

$$\rho_0 = \lambda_0 \delta(y) \delta(z - vt) \tag{3}$$

$$j = \rho v \hat{z} \tag{4}$$

where $\hat{z}$ is the unit vector in the positive z-direction. The electric and magnetic fields $\boldsymbol{E}$ and $\boldsymbol{B}$ are expressed in terms of the scalar and vector potentials ϕ and $\boldsymbol{A}$

$$\boldsymbol{E} = -\operatorname{grad}\phi - \frac{\partial \boldsymbol{A}}{\partial t} \tag{5}$$

$$\boldsymbol{B} = \operatorname{curl}\boldsymbol{A}. \tag{6}$$

Then, rewriting Maxwell's equations based on the condition

$$\epsilon_y^*\epsilon_0\frac{\partial\phi}{\partial t} + \frac{1}{\mu^*\mu_0}\frac{\partial A_z}{\partial z} = 0 \tag{7}$$

we can obtain the following equations for the potentials:

$$\epsilon_0\left(\epsilon_y^*\frac{\partial^2\phi}{\partial y^2} + \epsilon_z^*\frac{\partial^2\phi}{\partial z^2}\right) - \epsilon_y^*\epsilon_z^*\mu^*\epsilon_0^2\mu_0\frac{\partial^2\phi}{\partial t^2} = -\lambda_0\delta(y)\delta(z-vt) \tag{8}$$

$$A_x = A_y = 0,\ A_z = \epsilon_y^*\mu^*\epsilon_0\mu_0 v\phi. \tag{9}$$

This condition corresponds to the Lorentz condition well known for the isotropic medium. The author calls this the modified Lorentz condition. We can take care of the fact that the coefficient of $\partial\Phi/\partial t$ is the permittivity in the y-direction perpendicular to the moving direction.

The relationship between the space and time of two systems of coordinates, one, S', in uniform motion in the positive z-direction with speed v relative to the other, S, is given by the Lorentz transformation (let the velocity of light in vacuum denote by c). Also, the four-potential $(\phi'/c, A'_{x'}, A'_{y'}, A'_{z'})$ in the moving frame S' and the four-potential $(\phi/c, A_x, A_y, A_z)$ in the rest frame S are related by the Lorentz transformation: $A'_{x'} = A'_{y'} = A'_{z'} = 0$ in this problem.

Applying the Lorentz transformation to (8) and using the property of delta function, $\delta(kx) = \delta(x)/|k|$, we can get

$$\epsilon_y^*\frac{\partial^2\phi}{\partial y'^2} + \epsilon_z^*\gamma^2\left(1-\epsilon_y^*\mu^*\beta^2\right)\frac{\partial^2\phi}{\partial z'^2} = -\frac{\lambda_0\gamma}{\epsilon_0}\delta(y')\delta(z') \tag{10}$$

where $\beta = v/c$ and $\gamma = 1/\sqrt{1-\beta^2}$. Using the relation between ϕ in S and ϕ' in S'

$$\phi = \gamma\phi' \tag{11}$$

we can rewrite (10) as follows:

$$\epsilon_{y'}^*\frac{\partial^2\phi'}{\partial y'^2} + \epsilon_{z'}^*\frac{\partial^2\phi'}{\partial z'^2} = -\frac{\lambda_0}{\epsilon_0}\delta(y')\delta(z') \tag{12}$$

where

$$\epsilon_{y'}^* = \epsilon_y^*,\ \epsilon_{z'}^* = \epsilon_z^*\left(1-\epsilon_y^*\mu^*\beta^2\right)\gamma^2. \tag{13}$$

Now, the source line charge λ_0 rests on the original point in S' and reversely, the medium moves uniformly with speed v in the negative z-direction. Therefore, we can derive ϕ' as the solution of the potential for the electrostatic problem in which the source line charge λ_0 is put on the original point in the anisotropic dielectric medium of the y'-direction and z'-direction relative dielectric constants, $\epsilon_{y'}^*$ and $\epsilon_{z'}^*$, respectively. We already know this solution in the previous paper [21]. It is

$$\phi' = \frac{\lambda_0}{2\pi\epsilon_0\sqrt{\epsilon_{y'}^*\epsilon_{z'}^*}}\ln\frac{b}{\sqrt{\dfrac{y'^2}{\epsilon_{y'}^*} + \dfrac{z'^2}{\epsilon_{z'}^*}}} \tag{14}$$

where b is an arbitrary constant.

We can represent ϕ by using the Lorentz transformation and (11), (13), and (14) as follows:

$$\phi = \frac{\lambda_0}{2\pi\epsilon_0\sqrt{\epsilon_y^*\epsilon_z^*\left(1-\epsilon_y^*\mu^*\beta^2\right)}} \cdot \ln\frac{b}{\sqrt{\dfrac{y^2}{\epsilon_y^*} + \dfrac{(z-vt)^2}{\epsilon_z^*\left(1-\epsilon_y^*\mu^*\beta^2\right)}}}. \tag{15}$$

Therefore, $\boldsymbol{E}$ and $\boldsymbol{B}$ can be obtained by substituting these ϕ and $\boldsymbol{A}(A_x = A_y = 0, A_z \neq 0)$ into (5) and (6), respectively, as follows:

$$\begin{cases} E_x = 0 \\ E_y = \dfrac{\lambda_0\epsilon_z^*\eta^2 y}{2\pi\epsilon_0\eta\sqrt{\epsilon_y^*\epsilon_z^*}\left\{\eta^2\epsilon_z^*y^2 + \epsilon_y^*(z-vt)^2\right\}} \\ E_z = \dfrac{\lambda_0\epsilon_y^*\eta^2(z-vt)}{2\pi\epsilon_0\eta\sqrt{\epsilon_y^*\epsilon_z^*}\left\{\eta^2\epsilon_z^*y^2 + \epsilon_y^*(z-vt)^2\right\}} \end{cases} \tag{16}$$

and

$$\begin{cases} B_x = \dfrac{-\mu^*\mu_0\lambda_0 v\eta^2 y\sqrt{\epsilon_y^*\epsilon_z^*}}{2\pi\eta\left\{\eta^2\epsilon_z^*y^2 + \epsilon_y^*(z-vt)^2\right\}} \\ B_y = 0 \\ B_z = 0 \end{cases} \tag{17}$$

where

$$\eta^2 = 1 - \epsilon_y^*\mu^*\epsilon_0\mu_0 v^2. \tag{18}$$

We see from (16) that the moving line charge emits electric flux in the radial direction.

III. Equivalent Metric and Normalized Metric Factors

From the form of (15) representing a scalar potential ϕ, let $(z-vt)$ denote by the new variable ζ

$$\zeta = z - vt. \tag{19}$$

We can consider the equivalent anisotropic medium with the following relative dielectric constants in the $y\zeta$-coordinates:

$$\begin{cases} \epsilon_y^* = \epsilon_y^* \\ \epsilon_\zeta^* = \epsilon_z^*\eta^2. \end{cases} \tag{20}$$

The problem for ϕ in the $y\zeta$-coordinates looks the same to the electrostatic problem in the anisotropic dielectric medium shown in the previous paper [21]. Therefore, we can define, for the moving line charge problem, the equivalent metric factor m_{eq} in the radial direction with the angle θ_y from the y axis perpendicular to the moving direction as follows:

$$m_{\text{eq}}\left(\epsilon_y^*, \epsilon_z^*, \theta_y\right) = m\left(\epsilon_y^*, \epsilon_\zeta^*, \theta_y\right) \tag{21}$$

where m is the metric factor and is defined in [21], that is

$$m\left(\epsilon_y^*, \epsilon_\zeta^*, \theta_y\right) \equiv \sqrt{\frac{1}{\epsilon_y^*}\cos^2\theta_y + \frac{1}{\epsilon_\zeta^*}\sin^2\theta_y}. \tag{22}$$

Also, we define the equivalent normalized metric factor n_{eq} as the equivalent metric factor normalized by the equivalent metric factor in the direction with the angle α from the y axis, that is

$$n_{\text{eq}}\left(\epsilon_y^*, \epsilon_z^*, \theta_y, \alpha\right) \equiv \frac{m_{\text{eq}}\left(\epsilon_y^*, \epsilon_z^*, \theta_y\right)}{m_{\text{eq}}\left(\epsilon_y^*, \epsilon_z^*, \alpha\right)}. \tag{23}$$

The α in (23) is defined as the angle between the interface of different media and the y axis being one of principal axes. On the other hand, the α for the case of single medium is arbitrary.

We define the equivalent effective path length $d'_{\mathrm{eq}P_1P_2}$ between the point $P_1(y_1, z_1)$ and the point $P_2(y_2, z_2)$ as follows:

$$d'_{\mathrm{eq}P_1P_2} = n_{\mathrm{eq}}(\epsilon_y^*, \epsilon_z^*, \theta_y, \alpha)\, d_{P_1P_2} \tag{24}$$

where

$$d_{P_1P_2} = \sqrt{(y_1 - y_2)^2 + (z_1 - z_2)^2}\ (= \text{actual path length}) \tag{25}$$

and θ_y is the angle between the direction of the line P_1P_2 and the y axis. Using this equivalent effective path length, we can express the contribution ϕ to the scalar potential at the point $P_2(y_2, z_2)$ due to only the moving line charge λ_0 at the point $P_1(y_1, z_1)$ as follows:

$$\phi = \frac{\lambda_0}{2\pi\epsilon_0\sqrt{\epsilon_y^*\epsilon_\zeta^*}} \ln\left(\frac{1}{d'_{\mathrm{eq}P_1P_2}}\right). \tag{26}$$

IV. Moving Image Line Charges

Consider the problems in which a line charge λ_0 moves at an uniform velocity v parallel or perpendicular to the interface of two different media I, II. We can solve these problems by considering the moving image line charges, as shown in Fig. 1, whose positions, magnitudes, and velocities can be obtained by letting the electromagnetic fields satisfy the continuity conditions in the interface using (16)–(18).

For a perpendicular motion, we get

$$\frac{\sqrt{\epsilon_{2y}^*}}{\eta_2\sqrt{\epsilon_{2z}^*}} z_0'' = \frac{\sqrt{\epsilon_{1y}^*}}{\eta_1\sqrt{\epsilon_{1z}^*}} z_0 \tag{27}$$

$$\frac{\sqrt{\epsilon_{2y}^*}}{\eta_2\sqrt{\epsilon_{2z}^*}} v_2 = \frac{\sqrt{\epsilon_{1y}^*}}{\eta_1\sqrt{\epsilon_{1z}^*}} v_1 \tag{28}$$

$$K = \frac{\eta_1\sqrt{\epsilon_{1y}^*\epsilon_{1z}^*} - \eta_2\sqrt{\epsilon_{2y}^*\epsilon_{2z}^*}}{\eta_1\sqrt{\epsilon_{1y}^*\epsilon_{1z}^*} + \eta_2\sqrt{\epsilon_{2y}^*\epsilon_{2z}^*}} \tag{29}$$

where

$$\eta_i^2 = 1 - \epsilon_{iy}^*\mu_i^*\beta_i^2 \qquad (i = 1, 2). \tag{30}$$

For a parallel motion, we get

$$\frac{\eta_2\sqrt{\epsilon_{2z}^*}}{\sqrt{\epsilon_{2y}^*}} y_0'' = \frac{\eta_1\sqrt{\epsilon_{1z}^*}}{\sqrt{\epsilon_{1y}^*}} y_0 \tag{31}$$

$$v_2 = v_1 \tag{32}$$

$$K = \frac{\dfrac{\sqrt{\epsilon_{1y}^*\epsilon_{1z}^*}}{\eta_1} - \dfrac{\sqrt{\epsilon_{2y}^*\epsilon_{2z}^*}}{\eta_2}}{\dfrac{\sqrt{\epsilon_{1y}^*\epsilon_{1z}^*}}{\eta_1} + \dfrac{\sqrt{\epsilon_{2y}^*\epsilon_{2z}^*}}{\eta_2}}. \tag{33}$$

Therefore, we see that the field solution (the final solution) at the time t in the medium I can be calculated by summing the contribution of a moving source charge λ_0, the source electromagnetic field, and that of the moving image charge $K\lambda_0$, the reflected electromagnetic field, and the field solution in the medium II the moving image charge $(1-K)\lambda_0$, the refracted electromagnetic field.

We can check out that the electric flux emitted from the source point travels as such that its equivalent effective path length

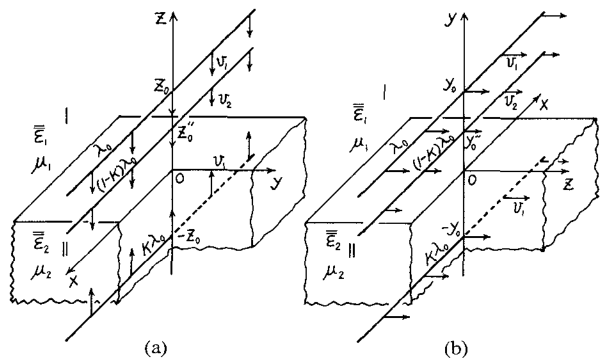

Fig. 1 Moving image line charges for a case of a moving line charges (a) Perpendicular motion. (b) Parallel motion.

becomes shortest similarly to the electrostatic case [21]. Therefore, we propose the following minimum principle of equivalent effective path length for electric flux of a moving line charge problem with anisotropic media which will be able to be solved by using the method of moving images:

$$\int_{\text{path}} n_{\mathrm{eq}}(\epsilon_y^*, \epsilon_z^*, \theta_y, \alpha)\, ds = \text{minimum}. \tag{34}$$

This principle states that the electric flux emitted from the moving source line charge chooses a trajectory that minimizes the equivalent effective path length. We must take note that this principle is valid for the refracted and reflected flux into which the final-electric flux associated with a moving single line charge at an arbitrary point in the region with anisotropic media is resolved, but not valid for the final-electric flux.

We can show the validity of this minimum principle by showing that we can obtain a trajectory of an electric flux emitted from the source point by solving the Euler equation for the principle (34) and then the image points can be obtained as points with the same equivalent effective path length to that from the point on the interface to the source point.

V. Conclusion

We have solved the moving line charge problem in the anisotropic dielectric medium. Applying those results, the moving line charge problem with two anisotropic dielectric media have been solved by the method of moving images. We have defined the equivalent metric factor and the equivalent normalized metric factor for an anisotropic medium. Using the equivalent normalized metric factor, we have shown the minimum principle of equivalent effective path length which shows us a trajectory for an electric flux to travel.

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Radial Line Transducer

E. SAWADO

I. Introduction

The purpose of this paper is to give a new excitation method of a radial-electromagnetic wave by a metallic cylinder with elliptical cross section. It was ascertained [1] that the radial wave propagates in a medium of permeability of quantity $\mu_\perp$. The quantity $\mu_\perp$ is given by $\mu_\perp=(\mu^2-\kappa^2)/\mu$, where μ and κ are the diagonal and nondiagonal components of the tensor permeability of a gyrotropic medium, respectively. The radial wave has interesting properties that this mode has not cutoff below the critical frequency $\omega=\gamma(BH)^{1/2}$, where ω is the angular frequency, $\gamma=1.76\times10^7$ ((oe s)$^{-1}$ in CGS unit), $B=\mu_0(H+M_s)$, the magnetic flux density, H the magnetic field, and M_s the saturation magnetization. Ganguly and Webb [2] presented an initial theory and some experiments for a magnetostatic surface wave single bar transducer. These investigations have concluded that the lowest operating frequency of Ganguly-type delay line is $\gamma(BH)^{1/2}$. Below this cutoff, no surface modes can exist. In view of the above, investigation of a radial wave type delay line should produce developments in low frequency microwave (0.5 to 1.5 GHz) applications.

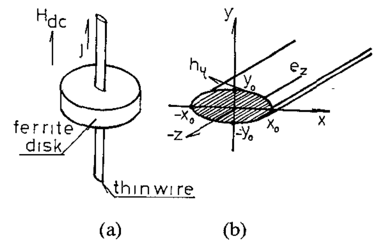

Fig. 1. Schematics for radial-line transducer with dimensions and coordinates. (a) Radial line with a fine sire of elliptical cross section (b) Diagram for a thin wire.

The system analyzed in this report is shown in Fig. 1. A transducer in the form of a fine wire with an elliptical cross section is excited with an RF current which generates radial volume waves within the structure. The dc magnetic field is directed along the z axis, and also the fine wire with an elliptical cross section is situated parallel to the z axis. This radial wave propagates perpendicular to the magnetic biasing fields, guided by parallel surfaces of a ferrite disk, and its energy is distributed within the medium. As Ganguly *et al.* pointed out previously, the frequency characteristics of the radiation resistance is influenced considerably by changing the microstrip width. The main subject of this paper is to demonstrate how a change of eccentricity of an elliptical metal cylinder influences the characteristics of the radiation resistance. The radiation pattern of this elliptic cylinder (Ribbon type) excitation possesses some type of directivity. If the eccentricity of the ellipse decreases, the patterns of the radiative power are highly directional, and this directed energy power tends to be confined to the direction of the y-axis shown in Fig. 1(b). It is possible to gain the maximum output power by placing the second thin wire within an area of maximum radiative power.

II. Basic Theory

When the fields are independent of z ($\partial/\partial z=0$), for the component e_z of the electric field vector, Maxwell's equation leads to the two-dimensional wave equation

$$\frac{\partial^2 e_z}{\partial x^2}+\frac{\partial^2 e_z}{\partial y^2}+\omega^2\epsilon\mu_0\mu_\perp e_z=0 \qquad (1)$$

where $\mu_\perp=\mu-\kappa^2/\mu$, μ and κ are a diagonal and a nondiagonal component of permeability tensor $\tilde{\mu}$, and μ_0 and ϵ are the vacuum permeability and dielectric constant, respectively. Applying the transformation defined by $x=h\cosh(\xi)\cos(\tau)$, $y=h\sinh(\xi)\sin(\tau)$, the wave equation leads to the equation

$$\frac{\partial^2 e_z}{\partial \xi^2}+\frac{\partial^2 e_z}{\partial \tau^2}+2k^2(\cosh(2\xi)-\cos(2\tau))e_z=0 \qquad (2)$$

where $2k=k_1h$, $k_1^2=\omega\epsilon^2\mu_0\mu$, $h=x_0(1-\tanh^2(\xi_0))^{1/2}$, and

$$\xi_0=0.5\ln\frac{(x_0+y_0)}{(x_0-y_0)}, \text{ (refer to Fig. 1 on } x_0 \text{ and } y_0).$$

The boundary condition is that, at the surface of a metal cylinder

$$e_z=0, \quad \xi=\xi_0. \qquad (3)$$

Here, for physically acceptable reasons, we assume that the fields having periods π, and the lowest mode $m=0$ is possible to excite, so that (6) has a nonzero value. Defining $q=(k_1h)^2/4$, the

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